

# Properties of the light quarks and antiquarks in the statistical approach

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## **Abstract**

In the quantum statistical parton distributions approach proposed more than one decade ago, one imposes relations between quarks and antiquarks expressions, which lead to very specific properties for the antiquarks. These properties have been verified up to now by recent data and it is a real challenge also for forthcoming experimental results, mainly in the high  $x$  region.

Let us now recall the main features of the statistical approach for building up the parton distributions function (PDFs). In this approach we treat simultaneously unpolarized distributions and helicity distributions, a unique situation in the literature.

The fermion distributions are given by the sum of two terms, a quasi Fermi-Dirac function and a helicity independent diffractive contribution:

$$xq^h(x, Q_0^2) = \frac{A_q X_{0q}^h x^{b_q}}{\exp[(x - X_{0q}^h)/\bar{x}] + 1} + \frac{\tilde{A}_q x^{\tilde{b}_q}}{\exp(x/\bar{x}) + 1} , \quad (1)$$

$$x\bar{q}^h(x, Q_0^2) = \frac{\bar{A}_q (X_{0q}^{-h})^{-1} x^{\bar{b}_q}}{\exp[(x + X_{0q}^{-h})/\bar{x}] + 1} + \frac{\tilde{A}_q x^{\tilde{b}_q}}{\exp(x/\bar{x}) + 1} , \quad (2)$$

at the input energy scale  $Q_0^2 = 1\text{GeV}^2$ . For the antiquarks we propose the ansatz (2) perfectly compatible with experimental data. We note that the diffractive term is absent in the quark helicity distribution  $\Delta q$ , in the quark valence contribution  $q - \bar{q}$  and in  $u - d$ .

In Eqs. (1,2) the multiplicative factors  $X_{0q}^h$  and  $(X_{0q}^{-h})^{-1}$  in the numerators of the non-diffractive parts of the  $q$ 's and  $\bar{q}$ 's distributions, imply a modification of the quantum statistical form, we were led to propose in order to agree with experimental data. The parameter  $\bar{x}$  plays the role of a *universal temperature* and  $X_{0q}^\pm$  are the two *thermodynamical potentials* of the quark  $q$ , with helicity  $h = \pm$ . They represent the fundamental parameters of the model. Notice the change of sign of the potentials and helicity for the antiquarks. For a given flavor  $q$  the corresponding quark and antiquark distributions involve the free parameters,  $X_{0q}^\pm$ ,  $A_q$ ,  $\bar{A}_q$ ,  $\tilde{A}_q$ ,  $b_q$ ,  $\bar{b}_q$  and  $\tilde{b}_q$ , whose number is reduced to *seven* by the valence sum rule,  $\int (q(x) - \bar{q}(x))dx = N_q$ , where  $N_q = 2, 1$  for  $u, d$ , respectively.

From a fit of unpolarized and polarized experimental data we have obtained for the potentials the values [1]:

$$X_u^+ = 0.475, \quad X_u^- = X_d^- = 0.307, \quad X_d^+ = 0.244. \quad (3)$$

To our surprise, it turns out that two potentials have identical numerical values, so for light quarks we have found the following hierarchy between the different potential components

$$X_u^+ > X_u^- = X_d^- > X_d^+. \quad (4)$$

We notice that quark helicity PDFs increases with the potential value, while antiquarks helicity PDFs increases when the potential decreases.

As a consequence of the above hierarchy it follows an hierarchy on the quarks helicity distributions,

$$xu_+(x) > xu_-(x) = xd_-(x) > xd_+(x) \quad (5)$$

and an obvious hierarchy for the antiquarks, namely

$$x\bar{d}_-(x) > x\bar{d}_+(x) = x\bar{u}_+(x) > x\bar{u}_-(x), \quad (6)$$

It is important to note that these inequalities Eqs. (5)-(6) are preserved by the next-to-leading QCD evolution, as we can see on Fig. 1 and Fig. 2, at least outside the diffractive region.

One important remark is that we have checked that the initial analytic form Eqs. (1,2), is almost preserved by the  $Q^2$  evolution with some small changes of the parameters.

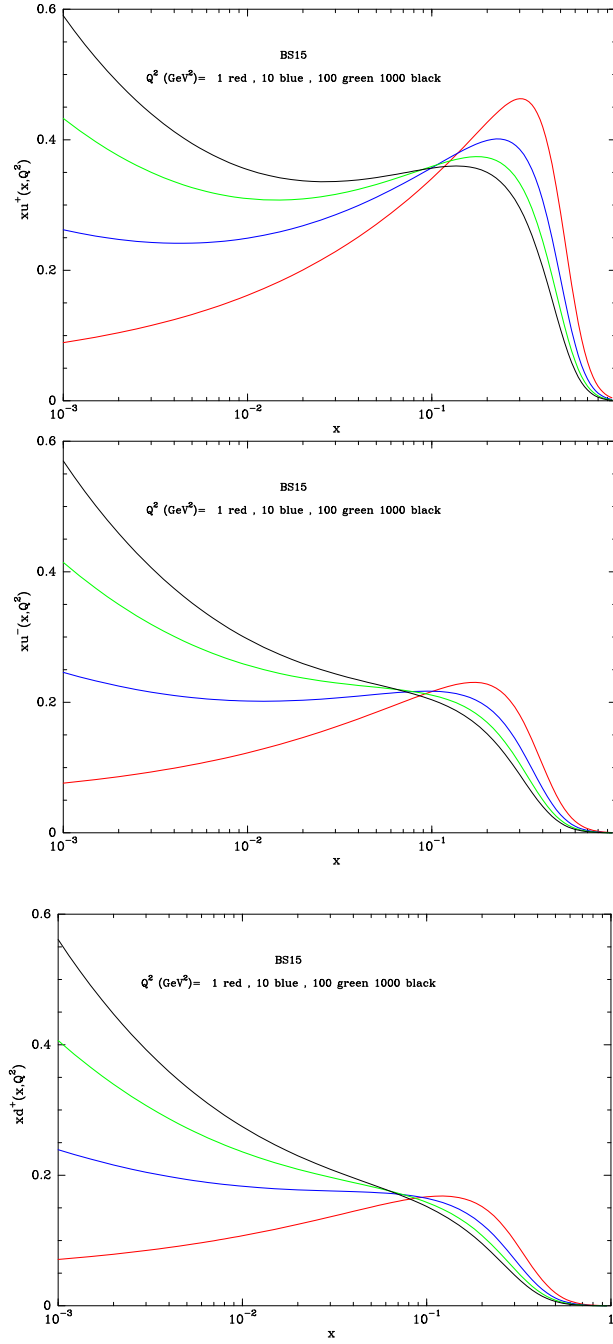


Figure 1: The different helicity components of the light quark distributions  $xf(x, Q^2)$  ( $f = u_+, u_- = d_-, d_+$ , from top to bottom), versus  $x$ , at  $Q^2 = 10, 100, 1000 \text{ GeV}^2$ , after NLO QCD evolution, from the initial scale  $Q^2 = 1 \text{ GeV}^2$ .

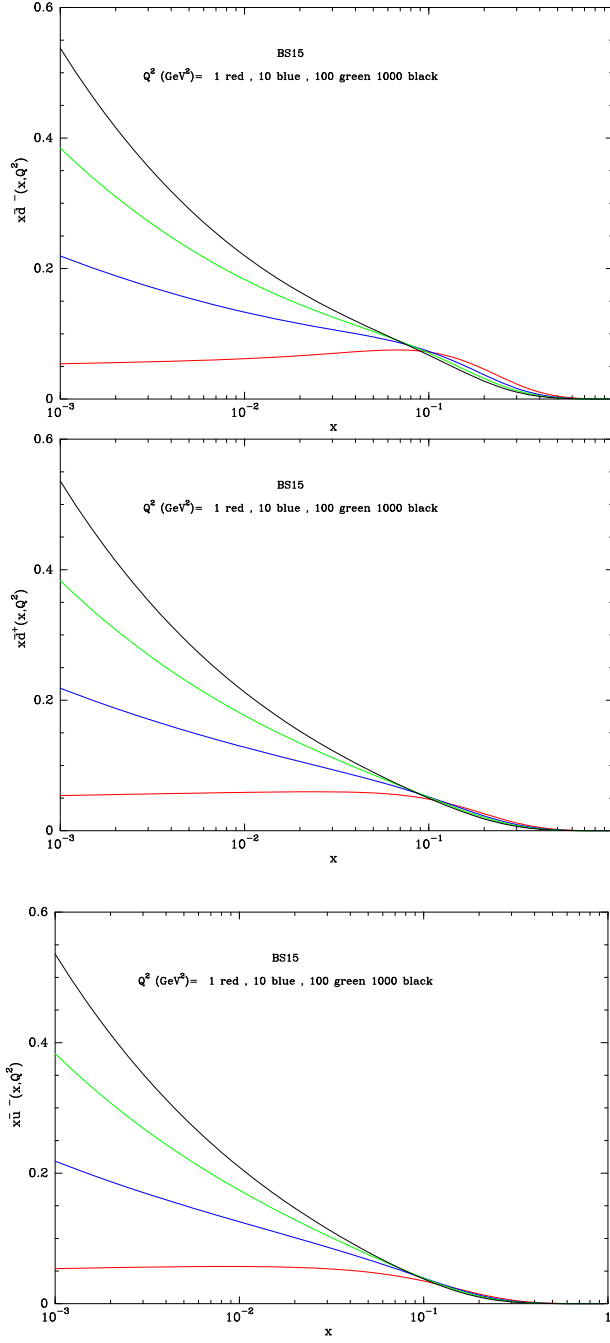


Figure 2: The different helicity components of the light antiquark distributions  $xf(x, Q^2)$  ( $f = \bar{d}_-, \bar{d}_+ = \bar{u}_+, \bar{u}_-$ , from top to bottom), versus  $x$ , at  $Q^2 = 10, 100, 1000 \text{ GeV}^2$ , after NLO QCD evolution, from the initial scale  $Q^2 = 1 \text{ GeV}^2$ .

One clearly concludes that  $u(x, Q^2) > d(x, Q^2)$  implies a flavor-asymmetric light sea, i.e.  $\bar{d}(x, Q^2) > \bar{u}(x, Q^2)$ , a trivial consequence of the Pauli exclusion principle, which is built in. Indeed this is based on the fact that the proton contains two  $u$  quarks and only one  $d$  quark.

Let us move on to mention more significant consequences concerning the helicity distributions which follow from Eqs. (3)-(6). First for the  $u$ -quark

$$x\Delta u(x, Q^2) > 0 \quad x\Delta\bar{u}(x, Q^2) > 0. \quad (7)$$

Similarly for the  $d$ -quark

$$x\Delta d(x, Q^2) < 0 \quad x\Delta\bar{d}(x, Q^2) < 0. \quad (8)$$

We have made these predictions almost 15 years ago [2] when, for simplifying reasons, it was more natural to assume that  $x\Delta\bar{u}(x, Q^2) = x\Delta\bar{d}(x, Q^2)$ .

Our predicted signs and magnitudes have been confirmed [1] by the measured single-helicity asymmetry  $A_L$  in the  $W^\pm$  production at BNL-RHIC from STAR [3].

Another important earlier prediction concerns the Deep Inelastic Scattering (DIS) asymmetries, more precisely  $(\Delta u(x, Q^2) + \Delta\bar{u}(x, Q^2))/(u(x, Q^2) + \bar{u}(x, Q^2))$  and  $(\Delta d(x, Q^2) + \Delta\bar{d}(x, Q^2))/(d(x, Q^2) + \bar{d}(x, Q^2))$ , shown in Fig. 3. Note that the data, so far, are in agreement with these predictions. In the high  $x$  region they differ from those which impose, for both quantities, the value one for  $x = 1$ . This is another challenge, since only up to  $x = 0.6$ , they have been measured at JLab [4].

There are two more important consequences which relate unpolarized and helicity distributions, namely for quarks

$$xu(x, Q^2) - xd(x, Q^2) = x\Delta u(x, Q^2) - x\Delta d(x, Q^2) > 0, \quad (9)$$

and similarly for antiquarks

$$x\bar{d}(x, Q^2) - x\bar{u}(x, Q^2) = x\Delta\bar{u}(x, Q^2) - x\Delta\bar{d}(x, Q^2) > 0. \quad (10)$$

This means that the flavor asymmetry of the light antiquark distributions is the same for the corresponding helicity distributions, as noticed long time ago [5].

Now let us come back to all these components  $xu_+(x, Q^2), \dots, x\bar{u}_-(x, Q^2)$  and more precisely to their  $x$ -behavior. It is clear that  $xu_+(x, Q^2)$  is the largest one and they are all monotonic decreasing functions of  $x$  at least for  $x > 0.2$ ,

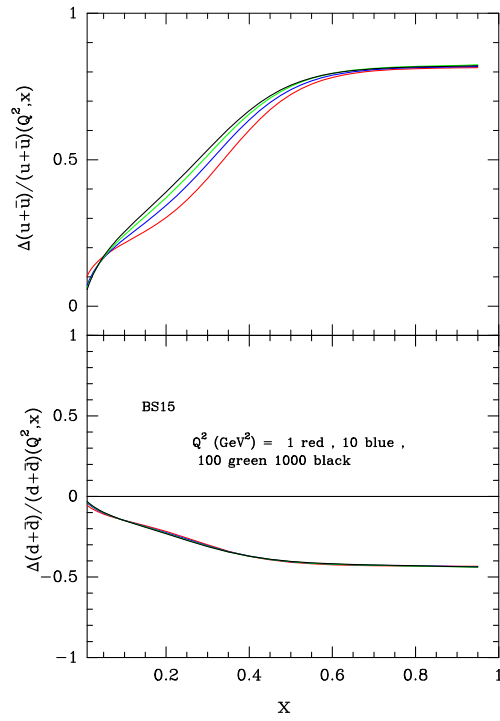


Figure 3: Predicted ratios  $(\Delta u(x, Q^2) + \Delta \bar{u}(x, Q^2))/(u(x, Q^2) + \bar{u}(x, Q^2))$  and  $(\Delta d(x, Q^2) + \Delta \bar{d}(x, Q^2))/(d(x, Q^2) + \bar{d}(x, Q^2))$ , versus  $x$ , at  $Q^2 = 10, 100, 1000 \text{ GeV}^2$ ,

outside the region dominated by the diffractive contribution see Figs. 1-2.

Similarly  $x\bar{d}_-(x, Q^2)$  is the largest of the antiquark components.

Therefore if one considers the ratio  $x\bar{d}(x, Q^2)/xu(x, Q^2)$ , its value is one at  $x = 0$ , because the diffractive contribution dominates and, due to the monotonic decreasing, it decreases for increasing  $x$ .

This falling  $x$ -behavior has been verified experimentally from the ratio of the DIS structure functions  $F_2^d/F_2^p$  and the charge asymmetry of the  $W^\pm$  production in  $\bar{p}p$  collisions [6].

Similarly if one considers the ratio  $x\bar{u}(x, Q^2)/x\bar{d}(x, Q^2)$ , its value is one at  $x = 0$ , because the diffractive contribution dominates and, due to the monotonic decreasing, it decreases for increasing  $x$ .

By looking at the curves ( See Figure 4), one sees similar behaviors. In both cases in the vicinity of  $x = 0$  one has a sharp behavior due to the fact that the diffractive contribution dominates and in the high  $x$  region there is a flattening out above  $x \simeq 0.6$ . It is remarkable to see that these ratios have almost no  $Q^2$  dependence.

To conclude we predict a monotonic increase of the ratio  $x\bar{d}(x, Q^2)/x\bar{u}(x, Q^2)$ . This was first observed in the low  $x$  region by the E866/NuSea collaboration [7] and very recently there is a serious indication from the preliminary results of the SeaQuest collaboration, that this trend persists beyond  $x = 0.2$  [8].

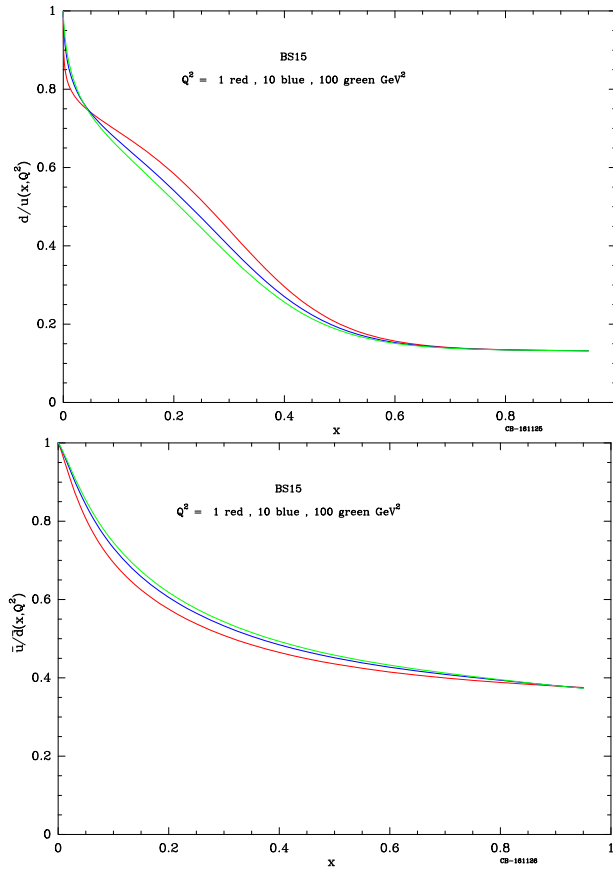


Figure 4: The ratios  $d(x, Q^2)/u(x, Q^2)$  (*top*) and  $\bar{u}(x, Q^2)/\bar{d}(x, Q^2)$  (*bottom*) versus  $x$  for different values of  $Q^2$ .

This prediction results from the following characteristic features of the statistical approach:

- 1) The hierarchy of the potentials Eq. (3) which are fundamental parameters in the approach.
- 2) The monotonic decreasing with  $x$  which is related to the Fermi-Dirac expression used to parameterise the parton distributions.
- 3) The expressions between quark and antiquarks we have supposed and which allow to relate the behavior of the ratios  $xd(x, Q^2)/xu(x, Q^2)$  and  $x\bar{u}(x, Q^2)/x\bar{d}(x, Q^2)$ .

Due to the high predictive power of our model it is a real challenge for several forthcoming data.



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